**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS: B**

As per data,

X =? (average time to repair car)

*μ* = 45 min

*σ* = 8 min

To finish in one hour, we must have X ≤ 50

find Pr (X > 50).

Pr (X > 50) = 1 –Pr (X ≤ 50).

Z = ( X-*μ* )/*σ* = (50-45)/8 = 0.625 = 73.4%

= 100-73.4 = 26.6%

Probability that the service manager will not meet his demand will be 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS:** Both the statements are True,

1. Let X = 44

*μ* = 38

*σ* =6

Probability of employees greater than age of 44 = Pr(X>44)

Z = (X-𝜇)/𝜎 = (44-38)/6 = Pr (Z ≤ 1) = 84.1345%

= 100 - 84.1345 = 15.86%

So the probability of number of employees between 38-44 years

of age = Pr(X<44) - 0.5 = 84.1345 - 0.5 = 34.1345%

(B) Let X = 30

*μ* = 38

*σ* =6

Probability of employees less than age of 30 = Pr(X<30)

Z = (X-𝜇)/𝜎 = (30-38)/6 = Pr (Z ≤ -1.33) = 9.12%

So the number of employees with probability 0.912 of them being

under age 30 = 0.0912\*400 =36.48 = 36 employees.

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS:**

We know that if X ∼N (μ1, σ1^2), and Y ∼N (μ2, σ2^2) are two

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independent random variables then,

X + Y ∼N (μ1 + μ2, σ1^2 +σ2^2)

X − Y ∼N (μ1 − μ2, σ1^2 + σ2^2)

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e.

Z is linear combination of X and Y then,

Z ∼N (aμ1 + bμ2, a^2σ1^2 + b^2σ2^2)

Therefore, 2X1~ N (2 u,4 σ^2) and X1+X2 ~

N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2 )2X1-(X1+X2) = N (4μ,6 σ^2)

2X1 will be greater scale version than X1 + X2. If X1 and X2

are normally distributed then the sum of the random sample

will be exactly same.

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS: D**

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values. Z = (X-μ) / σ.

For Probability 0.005 the Z Value is -2.57.

Z \* σ + μ = X

Z (-0.005) \* 20 + 100 = - (-2.57) \* 20 + 100 = 151.4

Z (+0.005) \* 20 + 100 = (-2.57) \* 20 + 100 = 48.6

Alpha= 99% for z-score alpha = 0.995 and 0.005

qnorm (0.995,100,20) = 151.5166

qnorm (0.005,100,20) = 48.48341

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45.
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company.
4. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS:**

A.

|  |  |  |  |
| --- | --- | --- | --- |
| stats.norm.ppf(0.975, loc=5, scale=3) | 10.87 | 45×10.87 | = 489.15 |
| stats.norm.ppf(0.25, loc=5,scale=3) | 2.97 | 45× 2.97 | = 133.65 |
| stats.norm.ppf(0.975, loc=7,scale=3) | 12.87 | 45×12.87 | = 579.15 |
| stats.norm.ppf (0.25, loc=7, scale=3) | 4.97 | 45× 4.97 | = 223.65 |

Range of Profit1 = (133.65, 489.15)

Range of Profit2 = (223.65, 579.15)

B.

|  |  |  |  |
| --- | --- | --- | --- |
| stats.norm.ppf(0.05, loc=5, scale=3) | 2.065 | 45×2.065 | = 92.925 |
| stats.norm.ppf(0.05, loc=7,scale=3) | 0.065 | 45×0.065 | = 02.925 |

5th percentile of profit in rupees = 92.925 + 02.925 = 95.25

C.

Second division a has larger probability of making a loss in

given year.